



# Explanation of the Characteristic Values of a Fiber Length Distribution

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## 1. Introduction

The purpose of this leaflet is to explain the characteristic values of a fiber length distribution in more detail. The scope and definition of the characteristic values are based on ISO 22314 "Plastics - Glass-fiber-reinforced products - Determination of fiber length". Although this document describes the manual evaluation of the fibers under the microscope, the section on the recommended content of an analysis report remains unaffected.

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## 2. Number of evaluated fibers, total fiber length, percentage of fiber pixels

These values do not yet give any indication of the shape of the length distribution. They rather help to estimate whether the evaluation was carried out with reasonable boundary conditions.

Thus, a certain number of fibers is necessary to obtain a representative distribution. The literature usually mentions values in the range of 1,000 to 2,000 fibers. However, for broad distributions of processed long fibers, 10,000 fibers or more are rather recommended.

For short fibers below 1 mm, this quantity is usually achieved with a single scan. For long fibers, several individual scans can be combined for evaluation.

The percentage of fiber pixels in the image is an indicator of whether the fiber dispersion has been sufficiently diluted. Too many fibers in the scan will result in many overlaps and multiple intersections where the program may take the wrong path while tracing a fiber. On the other hand, very heavy thinning with few fibers in the image is generally not a problem, as long as the number of evaluated fibers is still large enough. As a rule of thumb, the percentage of fiber pixels should not exceed 1-2 %.

## 3. Arithmetic mean value $L_n$

This is the usual average value, calculated from the total length of all fibers divided by their number:

$$L_n = \frac{\sum_{i=1}^n L_i}{n}$$

With  $L_i$  = Length of the i-th fiber

$n$  = Total number of fibers

#### 4. Weighted mean value $L_p$

The arithmetic mean responds strongly to dust fractions, since the short fiber fragments contribute little to the total length but much to the amount of fibers. Due to the squaring, the weighted average values long fibers more than short ones:

$$L_p = \frac{\sum_{i=1}^n L_i^2}{\sum_{i=1}^n L_i}$$

In ISO 22314, a formula is used which assumes that the fibers are already classified by length. In addition, the variable  $n$  is declared differently than in the arithmetic mean formula. The above, otherwise equivalent formula avoids these unnecessary changes.

#### 5. $L_p / L_n$

Ratio of weighted and arithmetic mean. For narrow distributions, which are common for short fibers, this ratio is usually close to 1. However, for wide long fiber distributions, values above 5 are also possible.

#### 6. Standard deviation

The standard deviation is calculated from the average squared deviation from the mean and is therefore a characteristic value for the width of the distribution:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (L_i - L_n)^2}{n}}$$

ISO 22314 explicitly refers to the standard deviation with the Greek letter sigma, not with a small "s". That means that the formula for the standard deviation of the population is meant, not that for a sample. In fact, however, we usually only take a section of the specimen to be examined and only a fraction of the fibers it contains are included in the evaluation. By definition, we therefore evaluate a small sample rather than the entirety of all fibers in the test specimen. However, the two formulas only differ in the denominator under the root, which is " $n$ " in one case and " $n-1$ " in the other. For many thousands of fibers, this difference is irrelevant.

## 7. Histogram

The purpose of the histogram is to graphically illustrate which fiber lengths are represented in the sample and to what extent. To do this, the length range found is divided into classes, i.e. into length sections of usually equal size. Each fiber is then assigned to the appropriate length range. Finally, you can choose between several alternatives for the visualisation:

- **Number:** On the Y-axis, the number of fibers is plotted for each length class.
- **Frequency:** On the Y-axis, the number of fibers contained in each length class is plotted as a percentage of the total number of fibers found.
- **Length ratio:** On the Y-axis, the total length of the fibers in each length class is plotted as a percentage of the total length of fibers found.

A second curve also shows the cumulative results of the classes. This makes it possible to see whether the few longest fibers are actually still shown in the histogram or whether the distribution stops beforehand. In the first case, the sum curve ends exactly at 100%, while in the second case a few percent are missing. The latter can be quite useful to exclude isolated overlong fibers, which often only result from two individual fibers touching at the ends, which are (incorrectly) interpreted as one continuous fiber.

## 8. Quantiles

These characteristic values do not appear in the ISO because the histogram already describes the length distribution as well as possible. However, there are situations when only single numerical values can be compared, e.g. in order to examine gradual improvements due to changes in the process settings.

In the case of a normal distribution, the mean value and standard deviation are sufficient to immediately draw the histogram. Fiber length distributions, however, are skewed and asymmetrical. Therefore, much more information is necessary to describe their shape. Quantiles, which are often also referred to as percentiles, help here.

To calculate a specific quantile, the fibers found are first sorted by length. After that, it depends on whether you want to know the quantiles of the frequency or the length ratio:

- **Frequency:** the x% quantile of the frequency describes at which fiber length x% of the total number of fibers is reached, if one starts at the shortest fiber and then counts up.

Example: 2978 fibers were found in the scan and the 25% quantile (d25) is to be calculated. 25% of 2978 is 744.5, so the length of the 745th fiber sorted in ascending order is the quantile we are looking for.

- **Length ratio:** the x% quantile of the length ratio describes at which fiber length x% of the total length of all fibers is reached, if one starts with the shortest fiber and then counts up.

Example: Fibers with a total length of 43,721  $\mu\text{m}$  were found in the scan and the 75% quantile (d75) is to be calculated. 75% of 43,721  $\mu\text{m}$  is 32,790.75  $\mu\text{m}$ . If the individual lengths of the sorted fibers are added up step by step, then the length of that fiber where the sum reaches or just exceeds 32,790.75  $\mu\text{m}$  is the quantile we are looking for.

Overall, the quantiles thus indicate at which fiber length a certain percentage of the total number or length of fibers is reached. From several, appropriately set quantiles (common are 1%, 5%, 10%, 25%, 50%, 75%, 90%, 95% and 99%), the shape of the histogram can thus be at least roughly estimated. For example, if a particularly high 10% quantile is found in several evaluations (whether in terms of frequency or length ratio), this indicates a higher dust content. Large differences between the 90% and 95% quantiles again indicate that the histogram does not slope steeply, but tapers off quite flatly to the right.

From the definition it is clear that the numerical value of a quantile must always correspond to the length of a fiber actually found. Especially with high quantiles above 90%, we often move into length ranges in which only a few fibers are still present. One long fiber more or less in the Petri dish can therefore lead to significant differences in the numerical value of the quantiles. Such differences should therefore not be overestimated.

## 9. Conclusion

Since the length of a fiber primarily determines its influence on the mechanical properties of the moulded part, the length-related characteristic values should preferably be displayed and rated. This applies to both the histogram and the associated quantiles.

The two final examples show histograms with marked characteristic values. This clearly shows that the typical asymmetrical shape of the fiber length distribution makes these much less descriptive compared to a normal distribution. For example, none of the mean values or the 50% quantiles (d50) represent the visually prominent peak of the curve..



